

THE EARLY HISTORY OF STRING THEORY AND SUPERSYMMETRY

■ JOHN H. SCHWARZ

*California Institute of Technology
Pasadena, CA 91125, USA*

Abstract

This lecture presents a brief overview of the early history of string theory and supersymmetry. It describes how the S-matrix theory program for understanding the strong nuclear force evolved into superstring theory, which is a promising framework for constructing a unified quantum theory of all forces including gravity. The period covered begins with S-matrix theory in the mid 1960s and ends with the widespread acceptance of superstring theory in the mid 1980s. Further details and additional references can be found in Schwarz (2007).

1 S-Matrix Theory

In UC Berkeley, where I was a graduate student in the mid 1960s, Geoffrey Chew (my thesis advisor), Stanley Mandelstam, and others focussed their efforts on constructing a theory of the strong nuclear force, *i.e.*, a theory of hadrons. Chew's approach to understanding the strong nuclear force was based on *S-matrix theory*. He argued that quantum field theory, which was so successful in describing QED, was inappropriate for describing a strongly interacting theory, where a weak-coupling perturbation expansion would not be useful. One reason for holding this view was that none of the hadrons seemed more fundamental than any of the others. Therefore a field theory that singled out some subset of the hadrons did not seem sensible. Also, it seemed impossible to formulate a quantum field theory with a fundamental field for every hadron. Chew spoke of *nuclear democracy* and the *bootstrap principle* to describe this situation. Chew advocated focussing attention on physical quantities, especially the S Matrix, which describes on-mass-shell scattering amplitudes. The goal was to develop a theory that would determine the hadron spectrum and hadronic S matrix.

The quark concept also arose during this period, but the prevailing opinion in the mid 1960s was that quarks are mathematical constructs, rather than physical entities, whose main use is as a mathematical technique for understanding symmetries and quantum numbers. The SLAC deep inelastic scattering experiments in the late 1960s made it clear that quarks and gluons are physical (confined) particles. It was then natural to try to base a quantum field theory on them, and QCD was developed a few years later with the discovery of asymptotic freedom. Thus, with the wisdom of hindsight, it is clear that Chew *et al.* were wrong to reject quantum field theory. Nonetheless, their insights were very influential, perhaps even

crucial, for the discovery of string theory, which can be regarded as the ultimate realization of the S-matrix theory program.

Some of the ingredients that went into the S-matrix theory program, such as unitarity and maximal analyticity of the S matrix, were properties (deduced from quantum field theory) that encode the requirements of causality and nonnegative probabilities. Another important ingredient was analyticity in angular momentum. The idea is that partial wave amplitudes $a_l(s)$, which are defined in the first instance for angular momenta $l = 0, 1, \dots$, can be extended to an analytic function of l , $a(l, s)$. The uniqueness of this extension results from imposing suitable asymptotic behavior in l . The Mandelstam invariant s is the square of the center-of-mass energy of the scattering reaction. The analytic function $a(l, s)$ can have isolated poles called *Regge poles*. (Branch points are also possible, but they are usually ignored.) The position of a Regge pole is given by a *Regge trajectory* $l = \alpha(s)$. A value of s for which $l = \alpha(s)$ takes a physical value corresponds to a physical hadron of spin l .

Theoretical work in this period was strongly influenced by experimental results. Many new hadrons were discovered in experiments at the Bevatron in Berkeley, the AGS in Brookhaven, and the PS at CERN. Plotting masses squared versus angular momentum (for fixed values of other quantum numbers), it was noticed that the Regge trajectories are approximately linear with a common slope

$$\alpha(s) = \alpha(0) + \alpha' s \quad \alpha' \sim 1.0 (\text{GeV})^{-2}.$$

Using the crossing-symmetry properties of analytically continued scattering amplitudes, one argued that exchange of Regge poles (in the t channel) controlled the high-energy, fixed momentum transfer, asymptotic behavior of physical amplitudes:

$$A(s, t) \sim \beta(t)(s/s_0)^{\alpha(t)} \quad s \rightarrow \infty, t < 0.$$

In this way one deduced from data that the intercept of the ρ trajectory, for example, was $\alpha_\rho(0) \sim .5$. This is consistent with the measured mass $m_\rho = .76 \text{ GeV}$ and the Regge slope $\alpha' \sim 1.0 (\text{GeV})^{-2}$.

The approximation of linear Regge trajectories describes long-lived resonances, whose widths are negligible compared to their masses. This approximation is called the *narrow resonance approximation*. In this approximation branch cuts in scattering amplitudes, whose branch points correspond to multiparticle thresholds, are approximated by a sequence of resonance poles. This is what one would expect in the tree approximation to a quantum field theory in which all the resonances appear as fundamental fields. However, there was also another discovery, called *duality*, which clashed with the usual notions of quantum field theory. In this context duality means that a scattering amplitude can be expanded in an infinite series of s -channel poles, and this gives the same result as its expansion in an infinite series of t -channel poles. To include both sets of poles, as usual Feynman diagram techniques might suggest, would amount to double counting.

2 The Discovery of String Theory

Veneziano (1968) discovered a simple analytic formula that exhibits duality with linear Regge trajectories. It is given by a sum of ratios of Euler gamma functions:

$$T = A(s, t) + A(s, u) + A(t, u), \quad \text{where} \quad A(s, t) = g^2 \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s) - \alpha(t))},$$

g is a coupling constant, and α is a linear Regge trajectory

$$\alpha(s) = \alpha(0) + \alpha' s.$$

The Veneziano formula gives an explicit realization of duality and Regge behavior in the narrow resonance approximation. The function $A(s, t)$ can be expanded as an infinite series of s -channel poles or of t -channel poles. The motivation for writing down this formula was largely phenomenological, but it turned out that formulas of this type describe scattering amplitudes in the tree approximation to a consistent quantum theory!

A generalization to incorporate adjoint $SU(N)$ quantum numbers was formulated by Paton and Chan (1969). Chan–Paton symmetry was initially envisaged to be a global (flavor) symmetry, but it was shown later to be a local gauge symmetry.

Very soon after the appearance of the Veneziano amplitude, Virasoro (1969) proposed an alternative formula

$$T = g^2 \frac{\Gamma(-\frac{1}{2}\alpha(s))\Gamma(-\frac{1}{2}\alpha(t))\Gamma(-\frac{1}{2}\alpha(u))}{\Gamma(-\frac{1}{2}\alpha(t) - \frac{1}{2}\alpha(u))\Gamma(-\frac{1}{2}\alpha(s) - \frac{1}{2}\alpha(u))\Gamma(-\frac{1}{2}\alpha(s) - \frac{1}{2}\alpha(t))},$$

which has similar virtues. Since this formula has total stu symmetry, it describes particles that are singlets of the Chan–Paton symmetry group.

Over the course of the next year or so, *dual models*, as the subject was then called, underwent a sudden surge of popularity, marked by several remarkable discoveries. One was the discovery (by several different groups) of an N -particle generalization of the Veneziano formula

$$A_N(k_1, k_2, \dots, k_N) = g_{\text{open}}^{N-2} \int d\mu_N(y) \prod_{i < j} (y_i - y_j)^{\alpha' k_i \cdot k_j},$$

where y_1, y_2, \dots, y_N are real coordinates. I will omit the description of the measure $d\mu_N(y)$, which can be found in Schwarz (2007). This formula has cyclic symmetry in the N external lines. Soon thereafter Shapiro (1970) formulated an N -particle generalization of the Virasoro formula:

$$A_N(k_1, k_2, \dots, k_N) = g_{\text{closed}}^{N-2} \int d\mu_N(z) \prod_{i < j} |z_i - z_j|^{\alpha' k_i \cdot k_j},$$

where z_1, z_2, \dots, z_N are complex coordinates. This amplitude has total symmetry in the N external lines.

Both of these formulas for multiparticle amplitudes were shown to have poles whose residues factorize in a consistent manner on an infinite spectrum of single-particle states.

This spectrum is described by a Fock space associated to an infinite number of harmonic oscillators

$$\{a_m^\mu\} \quad \mu = 0, 1, \dots, d-1 \quad m = 1, 2, \dots$$

where d is the dimension of Minkowski spacetime, which was initially assumed to be four. There is one set of such oscillators in the Veneziano case and two sets in the Shapiro–Virasoro case. These spectra were interpreted as describing the normal modes of a relativistic string: an open string (with ends) in the first case and a closed string (loop) in the second case. Amazingly, the formulas were discovered before this interpretation was proposed. In the above formulas, the y coordinates parametrize points on the boundary of a string world sheet, where particles that are open-string states are emitted or absorbed, whereas the z coordinates parametrize points on the interior of a string world sheet, where particles that are closed-string states are emitted or absorbed. (It is also possible to construct amplitudes in which both types of particles participate.)

Having found the factorization, it became possible to compute radiative corrections (loop amplitudes). Gross, Neveu, Scherk, and Schwarz (1970) discovered unanticipated singularities in a particular one-loop diagram for which the world sheet is a cylinder with two external particles attached to each of the two boundaries. The computations showed that this diagram gives branch points that violate unitarity. This was a very disturbing conclusion, since it seemed to imply that the classical theory does not have a consistent quantum extension. However, soon thereafter it was pointed out by Lovelace (1971) that these branch points become poles provided that

$$\alpha(0) = 1 \quad \text{and} \quad d = 26.$$

Prior to this discovery, everyone assumed that the spacetime dimension should be $d = 4$. We had no physical reason to consider extra dimensions. It was the mathematics that forced us in that direction. Later, these poles were interpreted as closed-string states in a one-loop open-string amplitude. Nowadays this is referred to as *open-string/closed-string duality*. This is closely related to *gauge/gravity duality*, which was discovered 27 years later.

The analysis also required there to be an infinite number of decoupling conditions, which turned out to coincide with the constraints proposed by Virasoro (1970) and further elaborated upon by Fubini and Veneziano (1971). Since the string has an infinite spectrum of higher-spin states, there are corresponding gauge invariances that eliminate unphysical degrees of freedom. The operators that describe the constraints that arise for a particular covariant gauge choice satisfy the Virasoro algebra

$$[L_m, L_n] = (m - n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m,-n},$$

where m, n are arbitrary integers. These operators can also be interpreted as generators of conformal symmetry for the two-dimensional string world sheet. The central charge (or conformal anomaly) c is equal to the spacetime dimension d . This anomaly cancels for $d = 26$ when the contribution of Faddeev–Popov ghosts is included.

3 The RNS Model and the Discovery of Supersymmetry

In a very inspired and important development, Ramond (1971) constructed a stringy analog of the Dirac equation, which describes a fermionic string. Just as the string momentum p^μ is the zero mode of a density $P^\mu(\sigma)$, where the coordinate σ parametrizes the string, he proposed that the Dirac matrices γ^μ should be the zero modes of densities $\Gamma^\mu(\sigma)$. Then he considered the Fourier modes of the dot product:

$$F_n = \int_0^{2\pi} e^{-in\sigma} \Gamma(\sigma) \cdot P(\sigma) d\sigma \quad n \in \mathbb{Z}.$$

In particular,

$$F_0 = \gamma \cdot p + \text{additional terms.}$$

He proposed that physical states of a fermionic string should satisfy the following analog of the Dirac equation

$$(F_0 + M)|\psi\rangle = 0.$$

He also observed that in the case of the fermionic string the Virasoro algebra generalizes to a super-Virasoro algebra

$$\{F_m, F_n\} = 2L_{m+n} + \frac{c}{3}m^2\delta_{m,-n}$$

$$[L_m, F_n] = \left(\frac{m}{2} - n\right)F_{m+n}$$

$$[L_m, L_n] = (m - n)L_{m+n} + \frac{c}{12}m^3\delta_{m,-n}.$$

Ramond's paper does not include the central extension, which turns out to be $c = 3d/2$, where d is the spacetime dimension. A little later, it was realized that consistency requires $d = 10$ and $M = 0$. These conditions are the analogs of $d = 26$ and $\alpha(0) = 1$ for the bosonic Veneziano string theory.

A couple of months later Neveu and Schwarz (1971a) constructed a new interacting bosonic string theory, which was called the *dual pion model*. It has a similar structure to the fermionic string, but the periodic density $\Gamma^\mu(\sigma)$ is replaced by an antiperiodic one $H^\mu(\sigma + 2\pi) = -H^\mu(\sigma)$. Then the Fourier modes, which differ from an integer by $1/2$,

$$G_r = \int_0^{2\pi} e^{-ir\sigma} H \cdot P d\sigma \quad r \in \mathbb{Z} + 1/2$$

satisfy a similar super-Virasoro algebra. Neveu and Schwarz (1971a) refers to this algebra as a *supergauge algebra*, a terminology that was sensible in the context at hand. The Neveu-Schwarz bosons and Ramond fermions were combined in a unified interacting theory of bosons and fermions by Neveu and Schwarz (1971b) and by Thorn (1971). This theory (the RNS model) was an early version of superstring theory. As will be explained shortly, a few crucial issues were not yet understood.

After a few more months, Gervais and Sakita (1971) showed that the the RNS model is described by the string world-sheet action

$$S = T \int d\sigma d\tau (\partial_\alpha X^\mu \partial^\alpha X_\mu - i\bar{\psi}^\mu \gamma^\alpha \partial_\alpha \psi_\mu),$$

where the coefficient T is the string tension. They also explained that it has *two-dimensional supersymmetry*, though that terminology was not used yet, by showing that it is invariant under the transformations

$$\delta X^\mu = \bar{\varepsilon} \psi^\mu, \quad \delta \psi^\mu = -i\gamma^\alpha \varepsilon \partial_\alpha X^\mu,$$

where ε is an infinitesimal constant spinor. To the best of my knowledge, this is the first supersymmetric theory identified in the literature! There are two possibilities for the world-sheet fermi fields ψ^μ . When it is antiperiodic $\psi^\mu = H^\mu$, which gives the boson spectrum (Neveu–Schwarz sector), and when it is periodic $\psi^\mu = \Gamma^\mu$, which gives the fermion spectrum (Ramond sector).

Five years later, Brink, Di Vecchia, and Howe (1976) and Deser and Zumino (1976) constructed a more fundamental world-sheet action with local supersymmetry. This formulation of the world-sheet theory has the additional virtue of also accounting for the super-Virasoro constraints. From this point of view, the significance of the super-Virasoro algebra is that the world-sheet theory, when properly gauge fixed and quantized, has *superconformal symmetry*. Again, the anomaly cancels for $d = 10$ when the Faddeev–Popov ghosts are included.

At about the same time as Ramond’s paper, the four-dimensional super-Poincaré algebra was introduced in a paper by Golfand and Likhtman (1971), who proposed constructing 4d field theories with this symmetry. This paper went unnoticed in the West for several more years. In fact, the celebrated paper of Wess and Zumino (1974), which formulated a class of 4d supersymmetric theories, was motivated by the search for 4d analogs of the 2d Gervais–Sakita world-sheet action. The Wess-Zumino paper launched the study of supersymmetric field theories, which proceeded in parallel with the development of supersymmetric string theory. Wess and Zumino (1974) used the expression *supergauge*, following the terminology of Neveu and Schwarz (1971), but in their subsequent papers they switched to *supersymmetry*, which was more appropriate for what they were doing.

4 The Temporary Demise of String Theory

String theory is formulated as an on-shell S-matrix theory in keeping with its origins discussed earlier. However, the SLAC deep inelastic scattering experiments in the late 1960s made it clear that the hadronic component of the electromagnetic current is a physical off-shell quantity, and that its asymptotic properties imply that hadrons have hard pointlike constituents. Moreover, all indications (at that time) were that strings are too soft to describe hadrons with their pointlike constituents.

By 1973–74 there were many good reasons to stop working on string theory: a successful and convincing theory of hadrons (QCD) was discovered, and string theory had severe problems as a theory of hadrons. These included an unrealistic spacetime dimension ($d = 10$ or $d = 26$), an unrealistic spectrum (including a tachyon and massless particles), and the absence of pointlike constituents. A few years of attempts to do better had been unsuccessful. Moreover, convincing theoretical and experimental evidence for the Standard Model was rapidly falling into place. That was where the action was. Even for those seeking to pursue speculative theoretical ideas there were options other than string theory that most people found more appealing, such as grand unification and supersymmetric field theory. Understandably, string theory fell out of favor. What had been a booming enterprise involving several hundred theorists rapidly came to a grinding halt. Only a few diehards continued to pursue it.

5 Gravity and Unification

Among the problems of the known string theories, as a theory of hadrons, was the fact that the spectrum of open strings contains massless spin 1 particles, and the spectrum of closed strings contains a massless spin 2 particle (as well as other massless particles), but there are no massless hadrons. In 1974, Joël Scherk and I decided to take string theory seriously as it stood, rather than forcing it to conform to our preconceptions. This meant abandoning the original program of describing hadron physics and interpreting the massless spin 2 state in the closed-string spectrum as a graviton. Also, the massless spin 1 states in the open-string spectrum could be interpreted as particles associated to Yang–Mills gauge fields. Specifically, Scherk and Schwarz (1974) proposed trying to interpret string theory as a unified quantum theory of all forces including gravity. Neveu and Scherk (1972) had shown that string theory incorporates the correct gauge invariances to ensure agreement at low energies (compared to the scale given by the string tension) with Yang–Mills theory. Yoneya (1973,1974) and Scherk and Schwarz (1974) showed that it also contains gauge invariances that ensure agreement at low energies with general relativity.

To account for Newton’s constant, the most natural choice for the fundamental string length scale was $l_s \sim 10^{-33}$ cm (the Planck length) instead of $l_s \sim 10^{-13}$ cm (the typical size of a hadron). Thus the strings suddenly shrank by 20 orders of magnitude, but the mathematics was essentially unchanged. The string tension is proportional to l_s^{-2} , so it increased by 40 orders of magnitude.

The proposed new interpretation had several advantages:

- Gravity and Yang–Mills forces are required by string theory.
- String theory has no UV divergences.
- Extra spatial dimensions could be a good thing.

Let me say a few words about the last point. In a nongravitational theory, the spacetime geometry is a rigid background on which the dynamics takes place. In that setup, the fact

that we observe four-dimensional Minkowski spacetime is a compelling argument to formulate the theory in that background geometry. As you know very well, this is part of the story of the Standard Model. However, in a gravitational theory that abides by the general principles laid out by Einstein, the spacetime geometry is determined by the dynamical equations. In such a setup extra dimensions can make sense provided that the equations of the theory have a solution for which the geometry is the product of four-dimensional Minkowski spacetime and a compact manifold that is sufficiently small to have eluded detection. It turns out that there are many such solutions. Moreover, the details of the compact manifold play a crucial role in determining the symmetries and particle content of the effective low-energy theory in four dimensions, even when the compact dimensions are much too small to observe directly.

6 Supersymmetry, Supergravity, and Superstrings

In the second half of the 1970s the study of supersymmetric field theories became a major endeavor. A few important supersymmetric theories that were formulated in that era included

- $\mathcal{N} = 1$, $d = 4$ supergravity, discovered by Freedman, Van Nieuwenhuizen, and Ferrara (1976) and Deser and Zumino (1976).
- $\mathcal{N} = 1$, $d = 10$ and $\mathcal{N} = 4$, $d = 4$ supersymmetric Yang–Mills theory discovered by Brink, Scherk, and Schwarz (1977) and Gliozzi, Scherk, and Olive (1977).
- $\mathcal{N} = 1$, $d = 11$ supergravity discovered by Cremmer, Julia and Scherk (1978).

Gliozzi, Scherk, and Olive (1976, 1977) proposed a truncation of the RNS string theory spectrum – *the GSO Projection* – that removes half of the fermion states and the “odd G-parity” bosons. In particular, the latter projection eliminates the tachyon. They showed that after the projection the number of physical bosonic degrees of freedom is equal to the number of physical fermionic degrees of freedom at every mass level. This was compelling evidence for *ten-dimensional spacetime supersymmetry* of the GSO-projected theory. Prior to this, we knew about the supersymmetry of the two-dimensional string world-sheet theory, but we had not considered the possibility of spacetime supersymmetry. In fact, the GSO projection is not just an option; it is required for consistency.

In 1979 Michael Green and I began a collaboration, which had the initial goal of understanding and proving the ten-dimensional spacetime supersymmetry of the GSO-projected version of the RNS theory. The highlights of our work included Green and Schwarz (1981, 1984a), which developed a new formalism in which the spacetime supersymmetry of the GSO-projected RNS string is manifest, and Green and Schwarz (1982), which classified the consistent ten-dimensional superstring theories and giving them the names Type I, Type IIA, and Type IIB. We were excited about these (and other) developments, but they did not arouse much interest in the theory community. String theory was still in the doldrums.

In the early 1980s there was growing interest in supersymmetry and extra dimensions. In particular, a small community became intrigued by Kaluza–Klein reduction of 11-dimensional

supergravity. Only the string ingredient was missing from their considerations. That changed following our next discovery.

7 Anomalies

If a unified theory is to make contact with the Standard Model, and have a chance of being realistic, parity violation is an essential ingredient. However, parity-violating classical theories generically have *gauge anomalies*, which means that they cannot be used to define quantum theories. The gauge symmetry is broken by one-loop quantum corrections, rendering the would-be quantum theory inconsistent. In the case of the Standard Model, if one were to change the theory by removing all of the leptons or all of the quarks, the theory would become inconsistent. When both the quarks and the leptons are included all gauge anomalies beautifully cancel, and so the Standard Model is a well-defined quantum theory. These considerations raise the question whether the potential gauge anomalies in chiral superstring theories also cancel, so that they give consistent quantum theories.

We knew that Type I superstring theory is a well-defined ten-dimensional theory at tree level for any $SO(n)$ or $Sp(n)$ gauge group, and that for every such group it is chiral (*i.e.*, parity violating). However, evaluation of a one-loop hexagon diagram in ten-dimensional super Yang–Mills theory, which describes the massless open-string states, exhibits explicit nonconservation of gauge currents, signalling a gauge anomaly. The only hope for consistency is that inclusion of the closed-string (gravitational) sector cancels this gauge anomaly without introducing new ones.

Type IIB superstring theory, which only has a closed-string gravitational sector, is also chiral and therefore potentially anomalous. It was not known how to analyze such anomalies until Alvarez-Gaumé and Witten (1984) derived general formulas for gauge, gravitational, and mixed anomalies in an arbitrary spacetime dimension. Using their results, they discovered that the gravitational anomalies, which would imply nonconservation of the stress tensor, cancel in Type IIB superstring theory. In their calculation this cancellation appears quite miraculous, though the UV finiteness of the Type IIB loop amplitudes implies that it had to work. Thus, Type IIB is a consistent chiral superstring theory. On the other hand, it did not look promising for describing the real world, since it does not contain any Yang–Mills gauge fields. (Many years later, nonperturbative Type IIB solutions that do contain Yang–Mills fields were discovered.) At that time, the last hope for constructing a realistic model seemed to reside with the Type I superstring theories, which are chiral and do contain Yang–Mills fields.

After a couple years of failed attempts, Green and I finally managed to compute the one-loop hexagon diagrams in Type I superstring theory. We found that both the cylinder and the Möbius-strip world-sheet diagrams contribute to the gauge anomaly and realized that there might be a gauge group for which the two contributions cancel. Green and Schwarz (1985) showed that $SO(32)$ is the unique choice for which the cancellation occurs. Since this

computation only demonstrated the cancellation of the pure gauge part of the anomaly, we decided to explore the low-energy effective field theory to see whether the gravitational and mixed anomalies also cancel. Using the results of Alvarez-Gaumé and Witten (1984), Green and Schwarz (1984b) verified that all gauge, gravitational, and mixed anomalies do in fact cancel for the gauge group $SO(32)$.

The effective field theory analysis showed that $E_8 \times E_8$ is a second (and the only other) gauge group for which the anomalies could cancel for a theory with $\mathcal{N} = 1$ supersymmetry in ten dimensions. In both cases, it is crucial for the result that the coupling to supergravity is included. The $SO(32)$ case could be accommodated by Type I superstring theory, but we didn't know a superstring theory with gauge group $E_8 \times E_8$. We were aware of the article by Goddard and Olive (1983) that pointed out (among other things) that there are exactly two even self-dual Euclidean lattices in 16 dimensions, and these are associated with precisely these two gauge groups. However, we did not figure out how to exploit this fact before the problem was solved by Gross, Harvey, Martinec, and Rohm (1985).

8 Epilogue

Following these discoveries there was a sudden surge of interest in superstring theory. After more than a decade, string theory had emerged from the doldrums. In my view, some of the new converts made a phase transition from being too pessimistic about string theory to being too optimistic about the near-term prospects for finding a realistic model. However, after a few years, almost all practitioners had a much more sober assessment of the challenges that remain. Superstring theory (including M-theory, which is part of the same theoretical framework) has remained a very active subject ever since 1984. Even though the construction of a complete and realistic model of elementary particles still appears to be a distant dream, the study of string theory has been enormously productive. For example, insights derived from these studies have had a profound impact on fundamental mathematics and are beginning to inspire new approaches to understanding topics in other areas of physics.

For many years string theory was considered to be a radical alternative to quantum field theory. However, in recent times – long after the period covered by this lecture – dualities relating string theory and quantum field theory were discovered. In view of these dualities, my current opinion is that string theory is best regarded as the logical completion of quantum field theory, and therefore it is not radical at all. There is still much that remains to be understood, but I am convinced that we are on the right track and making very good progress.

This work was supported in part by the U.S. Dept. of Energy under Grant No. DE-FG03-92-ER40701.

References

- Alvarez-Gaumé, L. & Witten, E. (1984). Gravitational anomalies. *Nuclear Physics B*, *234*, 269-330.
- Brink, L., Di Vecchia, P., & Howe, P. (1976). A locally supersymmetric and reparametrization invariant action for the spinning string. *Physics Letters B*, *65*, 471-474.
- Brink, L., Schwarz, J. H., & Scherk, J. (1977). Supersymmetric Yang-Mills theories. *Nuclear Physics B*, *121*, 77-92.
- Cremmer, E., Julia, B., & Scherk, J. (1978). Supergravity theory in eleven-dimensions. *Physics Letters B*, *76*, 409-412.
- Deser, S., & Zumino, B. (1976a). Consistent supergravity. *Physics Letters B*, *62*, 335-337.
- Deser, S., & Zumino, B. (1976b). A complete action for the spinning string. *Physics Letters B*, *65*, 369-373.
- Freedman, D. Z., Van Nieuwenhuizen, P., & Ferrara, S. (1976). Progress toward a theory of supergravity. *Physical Review D*, *13*, 3214-3218.
- Fubini, S., & Veneziano, G. (1971). Algebraic treatment of subsidiary conditions in dual resonance models. *Annals of Physics*, *63*, 12-27.
- Gervais, J. L., & Sakita, B. (1971). Field theory interpretation of supergauges in dual models. *Nuclear Physics B*, *34*, 632-639.
- Gliozzi, F., Scherk, J., & Olive, D. (1976). Supergravity and the spinor dual model. *Physics Letters B*, *65*, 282-286.
- Gliozzi, F., Scherk, J., & Olive, D. (1977). Supersymmetry, supergravity theories and the dual spinor model. *Nuclear Physics B*, *122*, 253-290.
- Goddard, P., & Olive, D. (1983). Algebras, lattices and strings. DAMTP-83/22. Reprinted in Goddard, P. (Ed.), & Olive, D. (Ed.): *Kac-Moody and Virasoro Algebras* (pp. 210-255). World Scientific 1988.
- Golfand, Yu. A., & Likhtman, E. P. (1971). Extension of the algebra of Poincaré group

- generators and violation of P invariance. *JETP Lett.*, *13*, 323-326.
- Green, M. B., & Schwarz, J. H. (1981). Supersymmetrical dual string theory. *Nuclear Physics B*, *181*, 502-530.
- Green, M. B., & Schwarz, J. H. (1982). Supersymmetrical string theories. *Physics Letters B*, *109*, 444-448.
- Green, M. B., & Schwarz, J. H. (1984a). Covariant description of superstrings. *Physics Letters B*, *136*, 367-370.
- Green, M. B., & Schwarz, J. H. (1984b). Anomaly cancellation in supersymmetric $D = 10$ gauge theory and superstring theory. *Physics Letters B*, *149*, 117-122.
- Green, M. B., & Schwarz, J. H. (1985). The hexagon gauge anomaly in Type I superstring theory. *Nuclear Physics B*, *255*, 93-114.
- Gross, D. J., Neveu, A., Scherk, J., & Schwarz, J. H. (1970). Renormalization and unitarity in the dual resonance model. *Physical Review D*, *2*, 697-710.
- Gross, D. J., Harvey, J. A., Martinec, E. J., & Rohm, R. (1985). The heterotic string. *Physical Review Letters*, *54*, 502-505.
- Lovelace, C. (1971). Pomeron form factors and dual Regge cuts. *Physics Letters B*, *34*, 500-506.
- Neveu, A., & Schwarz, J. H. (1971a). Factorizable dual model of pions. *Nuclear Physics B*, *31*, 86-112.
- Neveu, A., & Schwarz, J. H. (1971b). Quark model of dual pions *Physical Review D*, *4*, 1109-1111.
- Neveu, A., Schwarz, J. H., & Thorn, C. B. (1971). Reformulation of the dual pion model. *Physics Letters B*, *35*, 529-533.
- Neveu, A., & Scherk, J. (1972). Connection between Yang-Mills fields and dual models. *Nuclear Physics B*, *36*, 155-161.
- Paton, J. E., & Chan, H. (1969). Generalized Veneziano model with isospin. *Nuclear Physics B*, *10*, 516-520.

- Ramond, P. (1971). Dual theory for free fermions. *Physical Review D*, 3, 2415-2418.
- Scherk, J., & Schwarz, J. H. (1974). Dual models for non-hadrons. *Nuclear Physics B*, 81, 118-144.
- Schwarz, J. H. (2007). The early years of string theory: a personal perspective. arXiv:0708.1917 [hep-th].
- Shapiro, J. A. (1970). Electrostatic analog for the Virasoro model. *Physics Letters B*, 33, 361-362.
- Thorn, C. B. (1971). Embryonic dual model for pions and fermions. *Physical Review D*, 4, 1112-1116.
- Veneziano, G. (1968). Construction of a crossing-symmetric Regge-behaved amplitude for linearly rising Regge trajectories. *Nuovo Cimento A*, 57, 190-197.
- Virasoro, M. (1969). Alternative constructions of crossing-symmetric amplitudes with Regge behavior. *Physical Review*, 177, 2309-2311.
- Virasoro, M. (1970). Subsidiary conditions and ghosts in dual resonance models. *Physical Review D*, 1, 2933-2936.
- Wess, J., & Zumino, B. (1974). Supergauge transformations in four dimensions. *Nuclear Physics B*, 70, 39-50.
- Yoneya, T. (1973). Quantum gravity and the zero slope limit of the generalized Virasoro model. *Nuovo Cimento Letters*, 8, 951-955.
- Yoneya, T. (1974). Connection of dual models to electrodynamics and gravodynamics. *Progress in Theoretical Physics*, 51, 1907-1920.