

BLACK HOLES IN THE SUPERWORLD

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Abstract

Some aspects of black holes in supersymmetric theories of gravity are reviewed and some recent results outlined.

✧ Talk given by Sergio Ferrara.

1 Introduction

Black holes are perhaps the most mysterious and fascinating outcome of Einstein's theory of General Relativity (*A. Einstein, 1880-1952*). This theory was the result of a deep intuition on the implications of the equivalence principle, whilst trying to merge Newton's Law of gravitation with general relativistic covariance. Its mathematical formulation was then realized in terms of Riemannian geometry of space-time (*B. Riemann, 1826-1866*). Nowadays black holes are predicted by fundamental candidate theories of Quantum Gravity like Superstring or M-Theory and they are observed in the sky as relics of collapsing stars. They seem to encompass many of the mysteries of the evolution of our Universe from its creation to its final destiny, the so-called Big Crunch, or its eternal existence, namely an endless expansion.

Astrophysical black holes have huge masses, typically of the order of magnitude of the solar mass scale, 2×10^{30} Kg, while Quantum Gravity black holes have tiny masses, of the order of the Planck mass scale, namely 2×10^{-8} Kg. note that this is still much bigger than the typical mass of the atomic nuclei, that is from one to ten proton masses (the mass of a proton being 1.6×10^{-27} Kg).

Supergravity black holes are the black holes of the superworld [1]. Supersymmetry requires that they are extremal, that is that they have vanishing temperature, are marginally stable but carry Entropy. Actually, the black-hole Entropy makes a bridge between classical gravity and Quantum Gravity. In fact, we recall that the macroscopic definition of the black-hole entropy (Bekenstein – Hawking Entropy) [2, 3] connects its value to the black-hole horizon area A_H :

$$S_{BH}^{macro} = \frac{k_B c^3}{G \hbar} \frac{1}{4} A_H \quad (1.1)$$

The microscopic definition of the black-hole entropy, instead, relates its value to the number N_{mic} of microstates of the quantum system underlying the black hole, namely:

$$S_{BH}^{micro} = k_B \ln(N_{mic}) \quad (1.2)$$

Remarkably these formulae, computed with appropriate approximations, give the same result in Superstring Theory [4].

From now on, we generally use Natural Units, where $c = \hbar = G = k_B = 1$.

2 What is the Superworld?

In order to understand what Superworld is, one first has to introduce the notion of Superspace [6, 7]. This is a geometrical entity which extends the notion of Riemannian manifold to that of Supermanifold. A point on a D -dimensional Riemannian manifold \mathcal{M}_D , endowed with a Lorentz signature (*H. A. Lorentz, 1853-1928*), is identified by giving numerical coordinates x^μ , ($\mu = 1, \dots, D$). To identify a point in a Supermanifold we need, besides the coordinates x^μ , also a set of Grassmann (*H. Grassmann, 1809-1877*) anticommuting coordinates θ_α ($\alpha = 1, \dots, 2^{\lfloor D/2 \rfloor}$) with two basic properties:

1. $\theta_\alpha \theta_\beta = -\theta_\beta \theta_\alpha$
which implies nilpotency: $\theta_\alpha^2 = 0$;
2. They transform as spinors (*E. Cartan, 1869-1951, H. Weyl, 1885-1955*) under the action of the Lorentz group and their properties are related to modules of Clifford Algebras (*W. K. Clifford, 1845-1879*) and to the Spin Group, namely the universal covering group of the D -dimensional Lorentz group [5].

Superworld is the physical entity corresponding to the mathematical concept of supermanifold, whose environment is not ordinary space but superspace. The group of motion in Superspace is supersymmetry, as much as the group of motion in ordinary space-time is the Poincaré group (*H. Poincaré, 1854-1912*). An infinitesimal supersymmetry transformation with spinorial parameter ϵ_α acts on the coordinates of superspace as follows:

$$x^\mu \rightarrow x^\mu + i\bar{\epsilon}^\alpha (\gamma^\mu)_\alpha{}^\beta \theta_\beta \Leftrightarrow \delta x^\mu = i\bar{\epsilon}^\alpha (\gamma^\mu)_\alpha{}^\beta \theta_\beta \quad (2.1)$$

$$\theta_\alpha \rightarrow \theta_\alpha + \epsilon_\alpha \Leftrightarrow \delta \theta_\alpha = \epsilon_\alpha \quad (2.2)$$

where γ^μ is a matrix satisfying the Clifford Algebra and $\bar{\epsilon}$ denotes the Dirac conjugate spinor, namely $\bar{\epsilon} = \epsilon^\dagger \gamma^0$. Commuting twice the action of such transformations with parameters ϵ_1 and ϵ_2 respectively, one finds that x^μ undergoes an infinitesimal translation:

$$[\delta_1, \delta_2]x^\mu = 2i\bar{\epsilon}_2^\alpha (\gamma^\mu)_\alpha{}^\beta \epsilon_{1\beta}. \quad (2.3)$$

The supersymmetry algebra is a graded Lie algebra [8] (*S. Lie, 1842-1899*) with basic anticommutator: [9, 10, 11]

$$\{Q_\alpha, \bar{Q}_\beta\} = 2(\gamma^\mu C)_{\alpha\beta} p_\mu \quad (2.4)$$

where the supersymmetry generators Q_α are Majorana spinors (*E. Majorana, 1906-1938*) and C denotes the charge-conjugation matrix.

The supergroup associated to the supersymmetry algebra acts on a supermanifold, denoted by $\mathcal{M}_{b,f} \equiv \mathcal{M}_{D,2^{[D/2]}}$, where b and f denote the number of bosonic and fermionic coordinates respectively. the total (graded) dimension of a supermanifold is $b + f$. As we will see in the following, the maximal possible number of coordinates of superspace is $b_{max} = 11$, $f_{max} = 32$.

Actually, one can extend the super Lie algebra by introducing N supersymmetry generators $Q_{\alpha I}$ ($I = 1, \dots, N$) acting on an N -extended superspace labeled by N Grassmannian spinor coordinates $\theta_{\alpha I}$. The basic anti-commutators now become

$$\{Q_{\alpha I}, \bar{Q}_\beta^J\} = 2(\gamma^\mu C)_{\alpha\beta} p_\mu \delta_I^J \quad (2.5)$$

$$\{Q_{\alpha I}, Q_{\beta J}\} = \varepsilon_{\alpha\beta} Z_{IJ} \quad (2.6)$$

where Z_{IJ} are ‘‘central terms’’ which commute with all the rest of the superalgebra, including the Lorentz generators. It is precisely the presence of the central charges Z_{IJ} which makes it possible the existence of supersymmetric Black Holes, as will be shown in the next section.

The interaction in the superworld are described by supersymmetric theories. It is remarkable that such theories may encompass gauge interactions, in particular Yang–Mills theories [12, 6], as well as gravitational interactions. In the latter case, the corresponding theory is called supergravity [13, 14]. However these theories exist only for few values of the number N of supersymmetries and of the space-time dimension D [15, 16]. In particular, Super Yang–Mills theories in $D = 4$ require $1 \leq N \leq 4$ and at most they live in $D = 10$ [17]. On the other hand supergravity theories at $D = 4$ require $1 \leq N \leq 8$ and at most they live in $D = 11$ dimensions [18].

3 From Schwarzschild to Reissner–Nordström: The case of extremal Black Holes.

The celebrated Black-Hole solution of pure Einstein theory looks, in a chosen frame of spherical coordinates

$$ds_{\text{Schw}}^2 = - \left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \quad (3.1)$$

where M denotes the ADM mass, that is the total energy of the black-hole configuration. The naked singularity at $r = 0$ is covered by the *event horizon* at $r = 2M$. By event horizon we mean a surface where the gravitational redshift is infinite, that is where the time intervals undergo an infinite dilation with respect to a distant observer. This is also a singularity of the metric but it is only a *coordinate* singularity which can be removed with an appropriate choice of coordinates, while the singularity at $r = 0$ is a real singularity of the theory, that is independent of the reference frame.

The generalization of the Schwarzschild solution to an electrically charged black hole in the Einstein–Maxwell theory is given by the Reissner–Nordström black hole, whose metric reads:

$$ds_{\text{RN}}^2 = - \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} \right) dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} \right)^{-1} dr^2 + r^2 d\Omega^2 \quad (3.2)$$

Here M denotes the ADM mass and Q the electric charge of the space-time configuration. Such configuration can be easily generalized for dyonic configurations where also a magnetic charge P is present by replacing in (3.2) Q^2 with $Q^2 + P^2$. This metric exhibits two horizons, at

$$r_{\pm} = M \pm \sqrt{M^2 - Q^2} = M \pm r_0 \quad (3.3)$$

where r_+ corresponds to the event horizon and r_- to the Cauchy horizon, together with the physical singularity at $r = 0$.

In cosmology a Cosmic Censorship Principle is postulated (see for example [19]). An event horizon should always cover the singularity at $r = 0$, so that the singularity be not accessible to an observer external to the event horizon of the black hole. This can be rephrased by saying that no "naked" singularities can exist. For the Reissner–Nordström solution the Cosmic Censorship principle requires $M \geq Q$.

As shown by Steven Hawking, black holes obey laws which are formally the same as the laws of thermodynamics, after an appropriate identification of the quantum numbers of the solution is given. In particular, the thermodynamical properties of the black holes relate the area of the event horizon to the Entropy through the Bekenstein–Hawking formula (see eq. (1.1) in Natural Units):

$$S_{BH} = \frac{1}{4} A_H = \frac{1}{4} \pi R_+^2, \quad (3.4)$$

where R_+ is the event horizon r_+ for the Reissner–Nordström black hole, while it becomes an effective radius in the presence of other black-hole attributes such as angular momentum J and/or scalar charges Σ . For instance, in the presence of the latter $R_+^2 = r_+^2 - \Sigma^2 \leq r_+^2$.

The fact that a black hole continuously increases its horizon area can be interpreted as the second law of thermodynamics if we identify the black hole entropy as proportional to the horizon area, as pointed out by S. Hawking. Further support to this interpretation is given by the other laws of thermodynamics. In particular, the 0th law of thermodynamics states that for a system in equilibrium there is a quantity, the temperature, which is constant. An analogous constant quantity exists for a black hole at equilibrium, the so-called surface gravity that for the Reissner–Nordström black hole is

$$\kappa = \frac{c}{(r_+ + r_-)r_+ - Q^2} \quad (3.5)$$

where

$$c = \frac{1}{2}(r_+ - r_-) \quad (3.6)$$

It is then possible to identify the black-hole temperature T_{BH} as

$$T_{BH} = \frac{1}{2\pi}\kappa = \frac{c}{2S_{BH}} \quad (3.7)$$

The analogy is completed by rewriting the first law of thermodynamics:

$$dE = TdS + \text{work terms} \quad (3.8)$$

as

$$dM = T_{BH}dS_{BH} + \dots = \frac{\kappa}{2\pi} \frac{A_H}{4} + \dots \quad (3.9)$$

and observing that the third law of thermodynamics, stating that it is impossible to achieve $T = 0$ by a finite number of physical processes, can be rephrased as the impossibility to achieve $\kappa = 0$ by a finite number of physical processes.

The black hole which reaches the limit equilibrium temperature $\kappa = 0$ is called extremal. This corresponds to $c = 0$, that is to $r_+ = r_-$. For the Reissner–Nordström black hole, this happens when $M = |Q|$. A supergravity black hole is supersymmetric (BPS saturated) if its ADM mass M equals the highest skew-eigenvalue of the central charge matrix $Z_{IJ} = -Z_{JI}$ evaluated

at asymptotic infinity. In the presence of scalar charges Σ , the extremality condition allows both for supersymmetric and non-supersymmetric black-hole configurations.

For stationary but non-static solutions, that is rotating black holes of angular momentum J (Kerr–Newman black holes), the horizon radii become

$$r_{\pm} = M \pm \sqrt{M^2 - Q^2 - P^2 - J^2/M^2} \quad (3.10)$$

so that for a neutral spinning black hole (Kerr black hole) we reach extremality when $M^2 = J$, that is when the extremality parameter $a^* \equiv J/(GM^2) = 1$. Kerr black holes have been observed in our galaxy, in particular **GRS 1915+105** is the heaviest of the stellar black holes so far known [20] in the Milky Way Galaxy, with 10 to 18 times the mass of the Sun and a value of spin $J \simeq 10^{78} \hbar$. It was discovered on 15 August 1992. It is a nearly extremal black hole since in this case the extremality parameter is $a^* = 0.98 \simeq 1$. It has been argued that such black hole has an exact Conformal Field Theory dual [21].

4 Black Holes and Supersymmetry

One of the main properties of supergravity is the presence of scalar fields not minimally coupled to vector fields. The typical form of the Lagrangian of a set of electromagnetic field strengths $F^\Lambda = dA^\Lambda$ (enumerated by capital Greek indices Λ, Σ) in supergravity is of the form:

$$\mathcal{L} \propto g_{\Lambda\Sigma}(\varphi) F_{\mu\nu}^\Lambda F^{\Sigma|\mu\nu} + \Theta_{\Lambda\Sigma}(\varphi) \frac{1}{2} F_{\mu\nu}^\Lambda F_{\rho\sigma}^\Sigma \epsilon^{\mu\nu\rho\sigma} \quad (4.1)$$

where the couplings $g_{\Lambda\Sigma}$ and $\Theta_{\Lambda\Sigma}$ depend on a set of scalar fields enumerated by an index s . This has the implication that the Maxwell–Einstein black hole solution gets a non-trivial modification. In particular, the metric flow of the black hole towards the horizon $r = r_H$ is accompanied by trajectories of scalar-fields evolution from asymptotic infinity to the horizon:

$$\begin{aligned} \lim_{r \rightarrow \infty} \varphi^s(r) &= \varphi_\infty^s \in \mathcal{M} \\ \lim_{r \rightarrow r_H} \varphi^s(r) &= \varphi_{crit}^s \in \mathcal{M} \end{aligned} \quad (4.2)$$

The resulting analysis exploits the *attractor mechanism* [22]. Indeed scalar fields behave as dynamical systems which, in their evolution towards the

black-hole horizon of an extremal black hole, loose memory of their initial conditions (at $\varphi = \varphi_\infty$) approaching a critical point where the first derivative vanishes:

$$\begin{aligned}\lim_{r \rightarrow r_H} \varphi^s(r) &= \varphi_{crit}^s(Q) \\ \lim_{r \rightarrow \infty} \frac{d}{dr} \varphi^s(r) &= 0,\end{aligned}\tag{4.3}$$

and whose value only depends on the set of charges Q . Consistency of the solution implies that φ_{crit} is a critical point of an *effective black-hole potential* $V_{BH}(\varphi, Q)$:

$$\lim_{r \rightarrow \infty} \frac{\partial}{\partial \varphi^s} V_{BH}|_{\varphi=\varphi_{crit}} = 0.\tag{4.4}$$

Moreover, the Bekenstein–Hawking entropy-area formula becomes [24, 25]:

$$S_{BH} = \frac{1}{4} A_H = \pi V_{BH}(Q, \varphi_{crit}(Q)).\tag{4.5}$$

Note that the critical value of the scalar fields for extremal black holes, satisfying the attractor mechanism, is given only in terms of the electric and magnetic vector of charges Q , and this explains why the entropy only depends on Q and not on the initial values of the scalar fields φ_∞ .

The attractor mechanism allows to reduce the dynamical black-hole flow to a *first-order* evolution both for supersymmetric and non-supersymmetric extremal black holes [26, 27, 28, 29]. Indeed, the black-hole potential V_{BH} may be always written for extremal black holes as

$$V_{BH} = W^2 + 2\partial_s W \partial^s W\tag{4.6}$$

where $W(\varphi, Q)$ is known as the (*fake*) *superpotential*. There are several properties of W that make it important. First of all, in terms of W , the second order equations of motion of the theory reduce to a set of first order equations. Moreover W is invariant under the electric-magnetic duality group. It has a clear meaning in the context of the Hamilton–Jacobi theory, since it allows the interpretation of the flow as an Hamiltonian flow whose Hamilton characteristic function is actually simply related to W [30, 31].

The attractor mechanism allows to classify black-hole solutions, that is critical points of the black-hole potential, through the electric-magnetic duality symmetry of the theory. For each orbit of the duality symmetry, the

fake superpotential W has a different expression. In the particular case of supersymmetric black holes, one obtains $W = |Z|$ where $|Z|$ is the highest skew-eigenvalue of the central charge matrix Z_{IJ} . The duality orbits are modules of *groups of type E_7* , as requested by the Gaillard–Zumino analysis [23] combined with the attractor mechanism. The group E_7 appears in supergravity as the duality group of the maximally extended $N = 8$ theory in four dimensions, in its symplectic 56 dimensional module relating 28 electric to 28 magnetic charges. The orbits of the **56** module classify the black-hole solutions preserving different fractions of the original $N = 8$ supersymmetry. Moreover, E_7 controls the ultraviolet divergences of perturbation theory since it is anomaly free, and its arithmetic subgroups $G \subset E_7(\mathbb{Z})$ may encode the non-perturbative quantum corrections. It happens that all the duality groups of four dimensional supergravity theories with a number of supersymmetries $N < 8$ are groups of type E_7 , that is they have symplectic representations admitting a symmetric quartic invariant polynomial, but not a quadratic one [32]. As an example, we have presented in Table 1 the possible $N = 2$ choices

	G	R module	Primitive symm. inv.
$J_3^\mathbb{O}$	$E_{7(-25)}$	56	I_4
$J_3^\mathbb{H}$	$SO^*(12)$	32	I_4
$J_3^\mathbb{C}$	$SU(3, 3)$	20	I_4
$J_3^\mathbb{R}$	$Sp(6, \mathbb{R})$	$14'$	I_4
\mathbb{R}	$SL(2, \mathbb{R})$	4	I_4
$\mathbb{R} \oplus \Gamma_{1,n-1}, n \in \mathbb{N}$	$SL(2, \mathbb{R}) \times SO(2, n)$	$(2, 2 + n)$	I_4
CP^n	$U(1, n)$	$(1 + n)_\mathbb{C}$	I_2

Table 1: Supergravity sequence for $N = 2$ symmetric spaces.

of groups G of the $\frac{G}{H}$ symmetric spaces and their symplectic representations R [33, 34]. The first column identifies the scalar manifold whose isometry group is G is given. In particular, for the first four entries, they are named with the Jordan algebras J_3 over octonions, quaternions, complex and real numbers respectively, to which they are related. As we see in the last column, all the duality groups listed are of Type E_7 groups [35] except the last one, which has a primitive quadratic invariant polynomial. For $N \geq 3$ supergravity theories the analogous sequence is given in Table 2. Again, all the groups appearing in the sequence are of type E_7 except the first one, which admits

a primitive symmetric invariant polynomial of order 2.

N	G	\mathbf{R}
$N = 3$	$U(3, n)$	$(\mathbf{3} + \mathbf{n})$
$N = 4$	$SL(2, \mathbb{R}) \otimes SO(6, n)$	$(\mathbf{2}, \mathbf{6} + \mathbf{n})$
$N = 5$	$SU(1, 5)$	$\mathbf{20}$
$N = 6$	$SO^*(12)$	$\mathbf{32}$
$N = 8$	$E_{7(7)}$	$\mathbf{56}$

Table 2: The supergravity sequence for $N \geq 3$

5 Future directions of research

We are just at the beginning of the exploration of the beautiful intricacy given by supergravity black-hole physics, its group-theoretical structure and quantum perspectives. It is clear that much work and effort has to be done to unveil all the physics behind their structure which are emerging from supergravity considerations. In particular, we may mention few possible future directions of research:

- Extension of black-hole solutions to multi-center configurations, the classification of their orbits and the study of their dynamics, regarding their splitting behavior and their relation to the underlying stringy microstate counting.

- Clarification of the role of the group E_7 as far as quantum corrections are concerned.
- Inclusion of the Attractor Mechanism in the presence of higher derivative modifications of gravity, as suggested by superstring theory.
- The role of $N = 8$ black holes in a perturbatively finite theory of $N = 8$ supergravity.

Acknowledgements

The work of S.F. is supported by the ERC Advanced Grant n. 226455, “Supergravity, Quantum Gravity and Gauge Fields”. This work was supported in part by the MIUR-PRIN contract 2009-KHZKRX.

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