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The Origin and Status of Spontaneous Symmetry Breaking

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Abstract

From its inception in statistical physics to its role in the construction and in the development of the Brout-Englert-Higgs mechanism in quantum field theory, the notion of spontaneous symmetry breaking permeates contemporary physics. The discovery at the LHC of the BEH boson would confirm the mechanism and promote the quest for unified laws of nature. These topics are reviewed with particular emphasis on conceptual issues.

1 Introduction

Physics, as we know it, is an attempt to interpret the diverse phenomena as particular manifestations of general laws. This vision of a world ruled by *testable* laws is relatively recent. Essentially it started at the Renaissance and experienced a rapid development. The crucial ingredient was the inertial principle, initiated by Galileo (1564-1642), which essentially states that the uniform motion of a system does not affect the physics within the system and hence cannot be detected by an experiment performed within the system. This is a profound idea: the very fact that we do not feel such a motion confirms the universality of the Galilean physics approach to the understanding of nature in the sense that we ourselves are viewed as a physical system.

Starting from the inertial principle, Newton formulated at the end of the 17th Century the celebrated universal law of gravitation. He envisaged the world as composed of small interacting entities, which we now call elementary particles. In the 19th century, Maxwell established the

[1]Contribution to "Past , Present and Future", The Pontifical Academy of Science.

general laws of electromagnetism explaining electric and magnetic phenomena as well as the propagation of light. These laws were expressed in terms of a field, that is an object filling an extended region of space, propagating like a wave with the velocity of light and transmitting electric and magnetic interactions. The notions of particles and waves were unified in a subtle manner during the first decades of the 20th Century in Quantum Mechanics and the inertial principle was extended by Einstein to electromagnetism in the theory of Relativity. On the other hand the Newtonian law of gravitation was generalized by Einstein in 1915. The new theory of gravity, called General Relativity, opened to scientific investigation the cosmological expansion of the universe. These impressive developments in the first half of the 20th Century made it conceivable that all phenomena, from the atomic level to the edge of the visible universe, be governed solely by the known laws of classical general relativity and quantum electrodynamics, the quantum version of Maxwells electromagnetic theory .

Gravitational and electromagnetic interactions are long range interactions, meaning they are felt by objects, no matter how far they are separated from each other. But the discovery of subatomic structures revealed the existence of other fundamental interactions that are short range, being negligible at larger distance scales. In the beginning of the 60s, the theoretical interpretation of short range fundamental interactions seemed to pose insuperable obstacles.

It is the notion of spontaneous symmetry breaking (SSB) as adapted to gauge theory that provided the clue for solving the problem.

This notion finds its origin in the statistical physics of phase transitions [1]. There, the low temperature ordered phase of the system can be asymmetric with respect to the symmetry principles that govern its dynamics. This is not surprising since more often than not energetic considerations dictate that the ground state or low lying excited states of a many body system become ordered. A collective variable such as magnetization picks up expectation value, which define an order parameter that otherwise would vanish by virtue of the dynamical symmetry (isotropy in the aforementioned example). More surprising was the discovery by Nambu in 1960 *[Nobel prize 2008]* that the vacuum and the low energy excitations of a relativistic field theory may bare the mark of SSB [2, 3]: The chiral symmetry of massless fermion fields is broken by a spontaneous generation of their mass. The breaking give rise to massless pseudoscalar modes, which Nambu identified with the massless limit of pion fields. In absence of massless *gauge fields* characteristic of hitherto known fundamental interactions, such *massless Nambu-Goldstone bosons* (NG) and the concomitant vacuum degeneracy are general features of spontaneous symmetry breaking of a continuous group. The occurrence of SSB, either of a continuous or a discrete group, is also marked by fluctuations of the order parameter described by generically *massive scalar bosons.*

Introducing the massless gauge fields renders local in space-time the otherwise global dynamical symmetry and leads to dramatic effects. While the massive scalar bosons survive, the massless NG bosons disappear as such but provide a longitudinal polarization for the gauge fields, which therefore become massive. The essential degeneracy of the vacuum is removed and local symmetry is preserved despite the gauge field masses.

This way of obtaining massive gauge fields and hence short-range forces out of a fundamental massless Yang-Mills gauge field Lagrangian was proposed in 1964 by Brout and Englert in quantum field theoretic terms [4] and then by Higgs in the equations of motion formulation [5]. Roughly this BEH mechanism *[R. Brout, F. Englert, P.W. Higgs - Wolf Prize 2004]* works as follows. We introduced scalar fields (i.e. having no spatial orientation) which acquire, in analogy with the ferromagnet, an average value pervading space. These scalar fields interact with a subset of long range forces, converting those to short range ones. We also showed that this mechanism can survive in absence of elementary scalar fields.

The preservation of local symmetry in the BEH mechanism makes the theory renormalizable, that is tames divergent quantum fluctuations. This was a feature of quantum electrodynamics but that is what was missing in previous failed attempts to cope with short range fundamental interactions. We suggested this property [6] in 1966 and its proof was achieved in the remarkable work of 't Hooft and Veltman [7] *[G. 't Hooft, M. Veltman - Nobel Prize 1999].* The renormalizability made entirely consistent the electroweak theory, proposed by Weinberg in 1967 *[S.L. Glashow, A. Salam, S. Weinberg - Nobel Prize 1979]*, related to a group theoretical model of Glashow and to the dynamics of the BEH mechanism.

The mechanism is well established by the discovery of the Z and W bosons in 1983 *[C. Rubbia, S. van der Meer - Nobel Prize 1984]* and by the detailed field theoretic computations confirming the electroweak theory within its suspected domain of validity. If the LHC discovers the massive scalar (BEH) boson of the electroweak theory, it would confirm the mechanism in its simplest form. More elaborate realizations of the BEH mechanism are possible, involving many such BEH bosons or new dynamics with composite scalars. Hopefully the LHC will tell.

The BEH mechanism thus unifies in the same consistent theoretical framework short- and long-range forces, became the cornerstone of the electroweak theory and opened the way to a modern view on unified laws of nature.

2 Spontaneous breaking of a global symmetry

2.1 Spontaneous symmetry breaking in phase transitions

Consider a condensed matter system, whose dynamics is invariant under a continuous symmetry acting globally in space and time. As the temperature is lowered below a critical one, the symmetry may be reduced by the appearance of an ordered phase. The breakdown of the original symmetry is always a discontinuous event at the phase transition point but the order parameters may set in continuously as a function of temperature. In the latter case the phase transition is second order. Symmetry breaking in a second order phase transition occurs in particular in ferromagnetism, superfluidity and superconductivity. I discuss here the ferromagnetic phase transition which illustrates three general features of global SSB: ground state degeneracy, the appearance of a "massless mode" when the dynamics is invariant under a continuous symmetry, and the occurrence of a "massive mode".

Below the Curie point T_C, in absence of external magnetic fields and of surface effects, the exchange potential between neighboring atomic spins induces in a ferromagnet a globally oriented magnetization. The dynamics of the system is clearly rotation invariant. This is SSB.

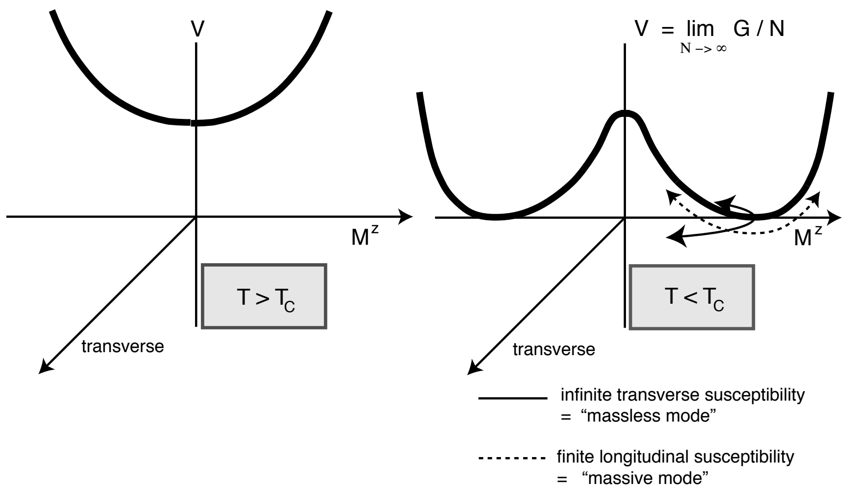

Figure 1: Effective potential of a typical ferromagnet.

The effective potential (i.e. the Gibbs free energy per spin) below the Curie point, depicted in

Fig.1, displays the essential features of SSB. At a given minimum, say, $\vec{M} = M^z \vec{1}_z$, the curvature of the effective potential measures the inverse susceptibility which determines the energy for infinite wavelength fluctuations, in other words, the "mass". The inverse susceptibility is zero in directions transverse to the order parameter and positive in the longitudinal direction. One thus gets, even at non-zero temperature, a "massless" transverse mode characteristic of broken continuous symmetry and we learn that there is also a (possibly unstable) "massive" longitudinal mode which corresponds to fluctuations of the order parameter. The latter mode is present in any spontaneous broken symmetry, continuous or even discrete. Such generically massive mode characterize any ordered structure, be it the broken symmetry phase in statistical physics, the vacuum of the global SSB in field theory presented in Section 2.2 or of the BEH mechanism discussed in Section 3. The mass of such longitudinal mode measures the rigidity of the ordered structure.

These general features of global SSB are common to nearly all second order phase transitions. However, in superconductivity, as a consequence of the long-range Coulomb interactions, the massless mode disappears by being absorbed by electron density oscillations, namely into a massive plasma mode [8, 9]. This effect can be viewed as a non-relativistic precursor of the BEH mechanism.

2.2 Broken continuous symmetry in field theory

Spontaneous symmetry breaking was introduced in relativistic quantum field theory by Nambu in analogy with the BCS theory of superconductivity. The problem studied by Nambu [2] and Nambu and Jona-Lasinio [3] is the spontaneous breaking of the chiral symmetry of massless fermions due to the invariance of the relative (chiral) phase between their decoupled right and left constituent neutrinos. Fermion mass cannot be generated perturbatively from a chiral invariant interaction but may arise dynamically from of a self-consistent fermion condensate. This breaks the chiral symmetry spontaneously.

The "massless mode" of SSB in phase transitions becomes a genuine *massless boson*, which is here a pseudoscalar boson coupled to the axiovector current. This is interpreted as the chiral limit of the (tiny on the hadron scale) pion mass. Such interpretation of the pion mass constituted a breakthrough in our understanding of strong interaction physics. The *massive scalar boson* measuring the rigidity of the condensate also occurs as a bound states of fermions.

Let us illustrate the occurrence of massless and massive SSB bosons in the simple model of a complex scalar field with $U(1)$ symmetry introduced by Goldstone [10]. The Lagrangian density,

$$\mathcal{L} = \partial^\mu \phi^* \partial_\mu \phi - V(\phi^* \phi) \quad \text{with} \quad V(\phi^*\phi) = -\mu^2 \phi^* \phi + \lambda(\phi^*\phi)^2 \quad , \quad \lambda > 0\,, \tag{2.1}$$

is invariant under the $U(1)$ group $\phi \to e^{i\alpha}\phi$. The global $U(1)$ symmetry is broken by a vacuum expectation value of the ϕ-field given, at the classical level, by the minimum of $V(\phi^*\phi)$. Writing $\phi = (\phi_1 + i\phi_2)/\sqrt{2}$, one may choose $\langle\phi_2\rangle = 0$. Hence $\langle\phi_1\rangle^2 = \mu^2/2\lambda$ and we select, say, the vacuum with $\langle\phi_1\rangle$ positive. The potential $V(\phi^*\phi)$ is depicted in Fig.2. It is similar to the effective potential below the ferromagnetic Curie point shown in Fig.1 and leads to similar consequences.

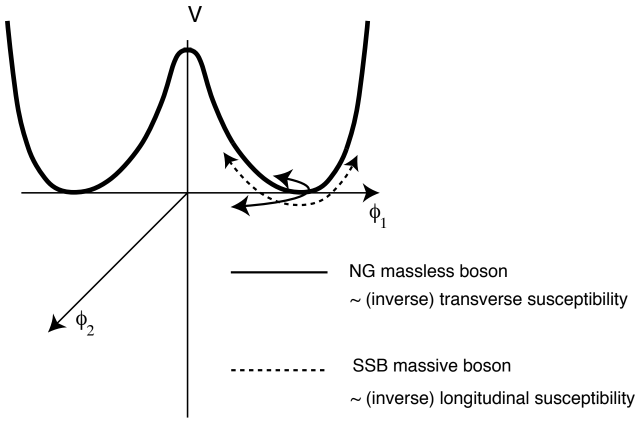

Figure 2: Spontaneous breaking of a continuous symmetry by scalar fields.

In the unbroken vacuum the field ϕ_1 has negative mass and acquires a positive mass $2\mu^2$ in the broken vacuum where the field ϕ_2 is massless. The latter is the *massless Nambu-Goldstone (NG) boson* of broken $U(1)$ symmetry and is the analog of the "massless mode" in ferromagnetism. The *massive scalar boson* describes the fluctuations of the order parameter $\langle\phi_1\rangle$ and is the analog of the "massive mode" in the ordered phase of a many-body system.

The origin of the massless NG boson is, as in the ferromagnetism phase, a consequence of the vacuum degeneracy. The vacuum characterized by the order parameter $\langle\phi_1\rangle$ is rotated into an equivalent vacuum by an operator proportional to the field ϕ_2 at zero space momentum. Such rotation costs no energy and thus the field ϕ_2 at space momenta $\vec{q} = 0$ has $q_0 = 0$, and hence is indeed massless.

3 The BEH mechanism

3.1 From global to local symmetry

The global $U(1)$ symmetry in Eq.(2.1) is extended to a local one $\phi(x) \to e^{i\alpha(x)}\phi(x)$ by introducing a vector field $A_\mu(x)$ transforming as $A_\mu(x) \to A_\mu(x) + (1/e)\partial_\mu\alpha(x)$. The corresponding Lagrangian density is

$$\mathcal{L} = D^\mu\phi^* D_\mu\phi - V(\phi^*\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\,, \tag{3.2}$$

with covariant derivative $D_\mu\phi = \partial_\mu\phi - ieA_\mu\phi$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

Local invariance under a semi-simple Lie group $\mathcal{G}$ is realized by extending the Lagrangian Eq.(3.2) to incorporate non-abelian Yang-Mills vector fields A_μ^a

$$\mathcal{L}_{\mathcal{G}} = (D^\mu\phi)^{*A}(D_\mu\phi)^A - V - \frac{1}{4}F_{\mu\nu}^a F^{a\,\mu\nu}\,, \tag{3.3}$$

$$(D_\mu\phi)^A = \partial_\mu\phi^A - eA_\mu^a T^{a\,AB}\phi^B \qquad F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - ef^{abc}A_\mu^b A_\nu^c\,. \tag{3.4}$$

Here, ϕ^A belongs to the representation of $\mathcal{G}$ generated by $T^{a\,AB}$ and the potential V is invariant under $\mathcal{G}$.

The local abelian or non-abelian gauge invariance of Yang-Mills theory hinges *apparently* upon the massless character of the gauge fields A_μ, hence on the long-range character of the forces they transmit, as the addition of a mass term for A_μ in the Lagrangian Eq.(3.2) or (3.3) destroys gauge invariance. But short-range forces such as the weak interaction forces, seem to be as fundamental as the electromagnetic ones despite the apparent departure from exact conservation laws. To reach a basic description of such forces one is tempted to link this fact to gauge fields masses arising from spontaneous broken symmetry. However the problem of SSB is very different for global and for local symmetries.

Consider the Yang-Mills theory defined by the Lagrangian Eq.(3.3). To exhibit the similarities and the differences between spontaneous breaking of a global symmetry and its local symmetry counterpart, it is convenient to choose a gauge which preserves Lorentz invariance and a residual global $\mathcal{G}$ symmetry. This can be achieved by adding to the Lagrangian a gauge fixing term $(2\eta)^{-1}\partial_\mu A^{a\mu}\,\partial_\nu A^{a\,\nu}$. The gauge parameter η is arbitrary and is not observable.

In such gauges the global symmetry can be spontaneously broken for suitable potential V by non zero expectation values $\langle\phi^A\rangle$ of scalar fields. In Fig.3 we have represented motions of this parameter in the spatial q-direction and in a direction B of the coset space $\mathcal{G}/\mathcal{H}$ where $\mathcal{H}$ is the unbroken subgroup. Fig.3a pictures the spontaneously broken vacuum of the gauge fixed

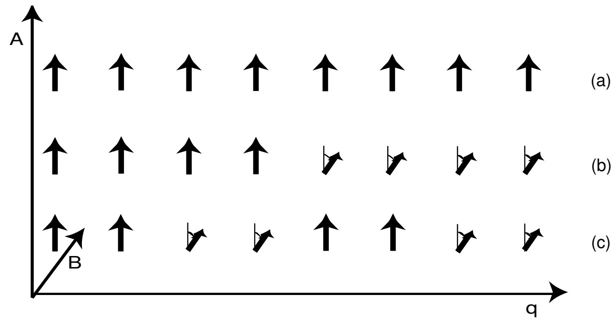

Figure 3: The disappearance of the massless NG boson in a gauge theory.

Lagrangian. Fig.3b and Fig.3c mimic motions in the coset with decreasing wavelength l. Clearly, as $l \to \infty$, such motions can only induce global rotations in the internal space. In absence of gauge fields, they would give rise, as in spontaneously broken global continuous symmetries, to massless NG modes generating the coset in the limit $l = \infty$. In a gauge theory, transverse fluctuations of $\langle \phi^A \rangle$ are just local rotations in the internal space and thus are unobservable gauge motions. Hence the would-be NG bosons induce only gauge transformations and their excitations disappear from the physical spectrum. A formal proof of the absence of the NG boson in gauge theories can be found in [11, 12] and will be further discussed in the following section.

But what makes local internal space rotations unobservable in a gauge theory is precisely the fact that they can be absorbed by the Yang-Mills fields. The absorption of the NG fields renders massive the gauge fields living in the coset $\mathcal{G}/\mathcal{H}$ by transferring to them their degrees of freedom which become longitudinal polarizations.

We shall see in the next sections how these considerations are realized in relativistic quantum field theory and give rise to vector masses in the coset $\mathcal{G}/\mathcal{H}$, leaving long-range forces only in a subgroup $\mathcal{H}$ of $\mathcal{G}$. Despite the unbroken local symmetry, the group $\mathcal{G}$ appears broken to its subgroup $\mathcal{H}$ in the asymptotic state description of field theory, and I shall therefore often term SSB such a Yang-Mills phase. The onset of SSB will now be described in detail mostly in lowest order perturbation theory around the self-consistent vacuum. This contains already the basic ingredients of the phenomenon.

3.2 The field theoretic approach

In this section, I will follow the method of reference [4]

α) Breaking by scalar fields

Let us first examine the abelian case as realized by the complex scalar field ϕ exemplified in Eq.(3.2). The interaction between the complex scalar field ϕ and the gauge field A_μ is

$$-ie\left(\partial_\mu\phi^*\phi - \phi^*\partial_\mu\phi\right)A^\mu + e^2 A_\mu A^\mu \phi^*\phi\,. \tag{3.5}$$

The SSB Yang-Mills phase is realized by a non vanishing expectation value for $\phi = (\phi_1 + i\phi_2)/\sqrt{2}$, which we choose to be in the ϕ_1-direction. Thus $\phi_1 = \langle\phi_1\rangle + \delta\phi_1$ and $\phi_2 = \delta\phi_2$; $\delta\phi_2$ and $\delta\phi_1$ are respectively as in Section 2.2 the NG massless boson and the massive scalar boson . In the covariant gauges, the free propagator of the field A_μ is

$$D^0_{\mu\nu} = \frac{g_{\mu\nu} - q_\mu q_\nu/q^2}{q^2} + \eta\frac{q_\mu q_\nu/q^2}{q^2}\,, \tag{3.6}$$

where η is the gauge parameter.

The polarization tensor $\Pi_{\mu\nu}$ of the gauge field in lowest order perturbation theory around the self-consistent vacuum is given by the tadpole graphs of Fig.4,

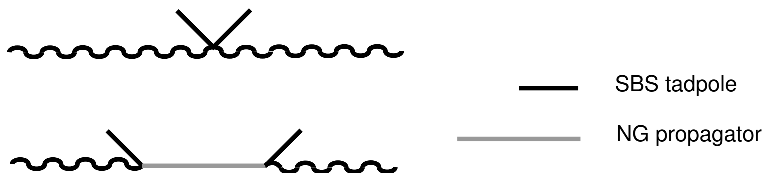

Figure 4: Tadpole graphs of SBS. Abelian gauge theory.

We see that, as a consequence of the contribution from the NG boson, the polarization tensor is transverse

$$\Pi_{\mu\nu} = (g_{\mu\nu}q^2 - q_\mu q_\nu)\Pi(q^2)\,, \tag{3.7}$$

and yields a singular polarization scalar $\Pi(q^2)$ at $q^2 = 0$

$$\Pi(q^2) = \frac{e^2\langle\phi_1\rangle^2}{q^2}\,. \tag{3.8}$$

From Eqs.(3.6), (3.7) and (3.8), the dressed gauge field propagator becomes

$$D_{\mu\nu} = \frac{g_{\mu\nu} - q_\mu q_\nu/q^2}{q^2 - \mu^2} + \eta\frac{q_\mu q_\nu/q^2}{q^2}\,, \tag{3.9}$$

which shows that the A_μ-field gets a mass

$$\mu^2 = e^2\langle\phi_1\rangle^2\,. \tag{3.10}$$

The transversality of the polarization tensor Eq.(3.7) results from the contribution of the NG boson and agrees with a Ward identity which guarantees that gauge invariance is preserved [6]. This means not only that the gauge field mass is gauge invariant but also that the gauge invariant vacuum is unbroken, as discussed in the previous section. Therefore there cannot be a NG boson in the physical spectrum.

The generalization of these results to the non abelian case described by the action Eq.(3.3) is straightforward. Writing the generators in terms of the real components of the fields, one gets the mass matrix

$$(\mu^2)^{ab} = -e^2 \langle \phi^B \rangle T^{a\,BC} T^{b\,CA} \langle \phi^A \rangle \,, \tag{3.11}$$

and the dressed gauge boson propagators have the same form as Eq.(3.9) in terms of the diagonalized mass matrix. As in the abelian case, the would-be NG bosons disappear from the physical spectrum and generate gauge invariant masses for the gauge fields in $\mathcal{G}/\mathcal{H}$. Long-range forces only survive in the subgroup $\mathcal{H}$ of $\mathcal{G}$ which leaves invariant the non vanishing expectation values $\langle \phi^A \rangle$.

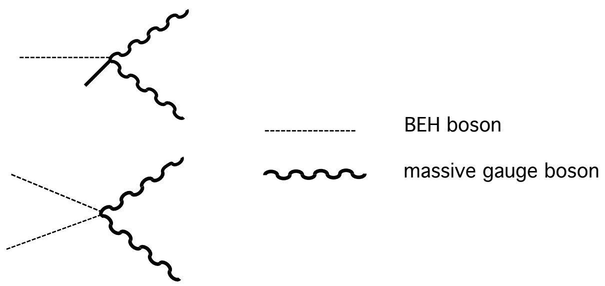

Figure 5: Coupling of BEH bosons to massive gauge bosons from Fig.4.

Note that the explicit form of the scalar potential V does not enter the computation of gauge field propagators which depend only on the expectation values at its minimum. This is because trilinear terms arising from covariant derivatives can only couple the tadpoles to the would-be NG bosons. Hence the massive scalar bosons decouple from the tadpoles in the gauge field propagators at the tree level considered here. Of course in the BEH mechanism the massive scalar (BEH) bosons couple to the massive gauge bosons already at the tree level. This is obvious from the diagrams of Fig 4 which defines the vertices coupling the BEH bosons to two massive gauge boson. These are depicted in Fig.5.

β) Dynamical symmetry breaking

The symmetry breaking giving mass to gauge vector bosons may arise from the fermion condensate. If a spontaneously global symmetry is extended to a local one by introducing gauge fields, the NG bosons are absorbed in massive gauge fields and disappear as such from the physical spectrum.

3.3 The renormalization issue

The interest in the BEH mechanism stems from the fact that it provides, as does quantum electrodynamics, a taming of quantum fluctuations. This allows the computation of the quantum effects necessary to cope with precision experiments. In other words, the theory is "renormalizable", in contradistinction to the theory of genuine non-abelian massive vector fields. A glimpse into this issue appears by comparing our field-theoretic approach [4] with the equation of motion approach of Higgs [5].

The massive vector propagator Eq.(3.9), which is also valid in the non-abelian case by diagonalizing the mass matrix Eq.(3.11), differs from a conventional free massive propagator in two respects. First the presence of the unobservable longitudinal term reflects the arbitrariness of the gauge parameter η. Second the NG pole at $q^2 = 0$ in the transverse projector $g_{\mu\nu} - q_\mu q_\nu / q^2$ is unconventional. Its significance is made clear by expressing the propagator of the A_μ field in Eq.(3.9) as (putting η to zero)

$$D_{\mu\nu} \equiv \frac{g_{\mu\nu} - q_\mu q_\nu / q^2}{q^2 - \mu^2} = \frac{g_{\mu\nu} - q_\mu q_\nu / \mu^2}{q^2 - \mu^2} + \frac{1}{\mu^2} \frac{q_\mu q_\nu}{q^2} \,. \tag{3.12}$$

The first term in the right hand side of Eq.(3.12) is the conventional massive vector propagator, while the second term is a pure gauge propagator due to the NG boson The decomposition Eq.(3.9) corresponds to the Higgs' transformation [5]

$$A_\mu = B_\mu + \frac{1}{e\langle \phi_1 \rangle} \partial_\mu \phi_2 \tag{3.13}$$

which absorbs explicitly the NG boson in a redefined gauged field B_μ which behaves as a conventional massive gauge vector field.

The propagator Eq.(3.9) which appears in the field theoretic approach contains thus, in the covariant gauges, the transverse projector $g_{\mu\nu} - q_\mu q_\nu / q^2$ in the numerator of the massive gauge field A_μ propagator. This is in sharp contradistinction to the numerator $g_{\mu\nu} - q_\mu q_\nu / \mu^2$ characteristic of the conventional massive vector field B_μ propagator. It is the transversality of the polarization tensor in covariant gauges, which led in the tree approximation to the transverse projector in Eq.(3.9). As mentioned above, the transversality of the polarization tensor is a consequence of a Ward identity and therefore does not rely on the tree approximation [6]. The importance of this fact is that transversality in covariant gauges determines the power counting of irreducible diagrams. It is then straightforward to verify that the quantum field theory formulation has the required power counting for a renormalizable field theory. On this basis it was suggested that it indeed was renormalizable [6].

However power counting is not enough to prove the renormalizability of a theory with local gauge invariance. To be consistent, the theory must also be unitary, a fact which is not apparent in "renormalizable" covariant gauges but is manifest in the "unitary gauge" defined in the free theory by the B_μ-field introduced in Eq.(3.13). In the unitary gauge however, power counting requirements fail. The equivalence between the A_μ and B_μ free propagators, *which is only true in a gauge invariant theory* where their difference is the unobservable NG propagator appearing in Eq.(3.12), is a clue of the consistency of the BEH theory. It is of course a much harder and subtler affair to proof that the full interacting theory is both renormalizable and unitary. This was achieved in the work of 't Hooft and Veltman [7], which thereby established the consistency of the BEH mechanism.

4 The electroweak theory and its BEH boson

I first review very briefly the basic elements of the electroweak theory.

In the electroweak theory for weak and electromagnetic interactions, the gauge group is taken to be $SU(2) \times U(1)$ with corresponding generators and coupling constants $gA_\mu^a T^a$ and $g' B_\mu Y'$. The $SU(2)$ acts on left-handed fermions only. The scalar field ϕ is a doublet of $SU(2)$ and its $U(1)$ charge is $Y' = 1/2$. Breaking is characterized by $\langle\phi\rangle = 1/\sqrt{2}\ \{0, v\}$ and $Q = T^3 + Y'$ generates the unbroken subgroup. Q is identified with the electromagnetic charge operator. The only residual massless gauge boson is the photon and the electric charge e is usually expressed in terms of the mixing angle θ as $g = e/\sin\theta, g' = e/\cos\theta$.

Using Eqs.(3.10) and (3.11) one gets the mass matrix

$$|\mu^2| = \frac{v^2}{4} \begin{vmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g'^2 & -gg' \\ 0 & 0 & -gg' & g^2 \end{vmatrix}$$

whose diagonalization yields the eigenvalues

$$M^2_{W^+} = \frac{v^2}{4} g^2 \qquad M^2_{W^-} = \frac{v^2}{4} g^2 \qquad M^2_Z = \frac{v^2}{4}\,(g'^2 + g^2) \qquad M^2_A = 0\,. \tag{4.14}$$

This permits to relate v to the the four Fermi coupling G, namely $v^2 = (\sqrt{2}G)^{-1}$.

The electroweak theory has been amply verified by experiment but the existence of its massive BEH boson presently search for at the LHC has, as yet, to be confirmed. Although the mechanism itself is well established by the discovery of the Z and W bosons and by the precision experiments, the discovery of the BEH boson would nevertheless constitute a direct proof of its validity. In addition, its properties would yield basic information which is crucial for further developments of elementary particle physics. First, one might get a better understanding of its structure, namely whether it would appear as a composite of higher energy elements or as an elementary object, in which case it might be related to supersymmetric multiplets: supersymmetry is indeed a natural framework for fundamental scalar bosons, which otherwise can easily arise as phenomenological constructs of complex structures. Second, the main content of the BEH mechanism is a consistent theory of charged Yang-Mills field, but its application to the electroweak theory is also used for generating all elementary fermion masses (a feature already possible from global SSB). The unification of such fermion mass generation with those of the gauge fields is an important experimental issue.

5 Concluding remarks

A prominent question is thus the existence of supersymmetry at the TeV scale for the reason just mentioned. In addition, supersymmetry would give more credence to searches for Grand Unification groups containing the subgroup $SU(2) \times U(1) \times SU(3)$ [13], where $SU(3)$ is the group of the strong interaction physics mediated by its quark confining gauge fields. Indeed, in minimal supersymmetric extensions of the Standard Model, renormalization group computations render more plausible the merging of these three groups at very high energies, namely at scales comparable with the expected onset of quantum gravity effects [14]. Supersymmetry and unification at such scales would favor the approach to quantum gravity by something akin to superstring theories.

These speculations have led to the unification paradigm whose ultimate realization would be a “theory of everything” including quantum gravity in the framework of some “M-theory”. However a word of caution is perhaps in order. Quite apart from the obvious philosophical questions raised by such quest in the present framework of theoretical physics, the transition from perturbative string theory to its elusive M-theory generalization hitherto stumbles on the treatment of non-perturbative gravity. This might well be a hint that new conceptual elements have to be found to cope with the relation between gravity and quantum theory and which may well be unrelated to a unification program.

Addendum

Since this paper was written, a dramatic event has occurred: the BEH boson has been found at the LHC at CERN and appears to be an elementary object (at the energy scale considered) consistent with the electroweak theory.

As discussed in Section 4, this provides a direct confirmation of the validity of the BEH mechanism. But more than that, the elementary character of the scalar boson appears to dispose of complicated dynamical schemes such as "extended technicolor" or "walking technicolor" needed when the simple "natural" technicolor scheme for generating gauge vector boson masses is extended to cope with dynamical elementary fermion masses. This is a welcome result but as pointed out in Section 4, it seems to suggest that (broken) supersymmetry be a likely generalization of the Standard Model. Although there is at present no experimental indication of such supersymmetric partners, we have to wait for further datas to ensure that supersymmetry is present or not at available energies. As pointed out in Section 5, in the latter case, the occurrence of supersymmetry at higher scale (and possibly only at scales close to the Planck scale) will in the forseeable future remain a purely speculative issue.

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1	F. Englert and R. Brout *Phys.Rev.Letters* **13-[9]** (1964) 321	26/06/1964	31/08/1964
2	P.W. Higgs *Phys. Letters* **12** (1964) 132	27/07/1964	15/09/1964
3	P.W. Higgs *Phys.Rev.Letters* **13-[16]** (1964) 508	31/08/1964	19/10/1964
4	G.S. Guralnik, C.R. Hagen and T.W.B. Kibble *Phys.Rev.Letters* **13-[20]** (1964) 585	12/10/1964	16/11/1964