Predictability in Science: Accuracy and Limitations
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CHANGE OF SCALING BEFORE EXTREME EVENTS IN COMPLEX SYSTEMS

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1. INTRODUCTION

Critical transitions. Natural and human-made complex systems persistently generate critical transitions – rare extreme events, also known as disasters, crises etc. Predictive understanding of critical transitions is commonly regarded as one of the major unsolved problems of basic science.

The dramatic practical aspect of that problem is disaster preparedness. Global population, economy, and environment become increasingly vulnerable to a throng of disasters, already threatening the very survival and sustainability of our civilization (Science for Survival and Sustainable Development, 2000; G8-UNESCO World Forum on 'Education, Innovation and Research: New Partnership for Sustainable Development', 2007).

This study explores a specific observable phenomenon, preceding critical transitions. It is a change of scaling – the size distribution of events comprising a process considered. Scaling itself is a staple of studying complexity. However premonitory change of scaling has been found so far only in seismicity and in other forms of multiple fracturing, with major failures (e.g.

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strong earthquakes) for critical transitions (e.g., Smith, 1986; Main *et al.*, 1989, 1992; Henderson *et al.*, 1992, 1994; Narkunskaya and Shnirman, 1994; Rotwain *et al.*, 1997; Wiemer and Wyss, 1997; Wyss and Wiemer, 2000; Burroughs and Tebbens, 2002; Amitrano, 2003; Zaliapin *et al.*, 2003).

Here we explore universality of this phenomenon, by uniform analysis of a variety of data: observations relevant to economic recessions, surges of unemployment, and homicides surges; and results of mathematical modeling. Altogether this seems to open the promising line of research in predicting critical transitions.

Next we outline the background of this study.

Predictability. Complex systems are not predictable in the Laplacean sense, with accuracy limited only by accuracy of data and theory. However, after a coarse graining, in a not-too-detailed scale, such systems exhibit regular behaviour patterns and become predictable, up to the limits. The holistic approach, 'from the whole to details' opens a possibility to overcome the complexity itself and the chronic imperfection of data as well (Farmer and Sidorowich, 1987; Ma *et al.*, 1990; Kravtsov, 1993; Gell-Mann, 1994; Holland, 1995; Kadanoff, 1976; Keilis-Borok and Soloviev, 2003).

Premonitory patterns. Dynamics of a complex system include a variety of observable processes; they show premonitory patterns emerging as an extreme event approaches (e.g., Keilis-Borok and Malinovskaya, 1964; Gabrielov *et al.*, 1986; Keilis-Borok, 1990; Newman *et al.*, 1995; Sammis *et al.*, 1996; Keilis-Borok and Shebalin, 1999; Sornette, 2000; Keilis-Borok and Soloviev, 2003). Note that while the targets of prediction – rare extreme events - have *low probability to occur but large impact on a system*, premonitory patterns are formed by more frequent events with *high probability to occur but low impact.*

Numerical modeling and data analysis demonstrated the *dual nature* of premonitory patterns: they are partly universal, as befits complexity, and partly process-specific. Change of scaling, explored here, is one of such patterns.

Difficulty and challenge of the predictability problem is also common for the frontiers of basic science: this is the absence of a complete set of fundamental equations governing formation of extreme events. A.N. Kolmogorov wrote:

> It became clear for me that it is unrealistic to have a hope for the creation of a pure theory [of the turbulent flows of fluids and gases] closed in itself. Due to the absence of such a theory we have to rely upon the hypotheses obtained by processing of the experimental data…

Pure theory is even less complete for dynamics of solid Earth (Keilis-Borok and Soloviev, 2003; Press, 1965) and – particularly – for socio-economic and political systems. Hence the bane of data processing – large number of adjustable parameters free for data-fitting.

New analytical framework. A possibility to reduce that number is recently provided by an exactly solvable model of diffusion with branching (Gabrielov *et al.*, 2007). This model analytically defines premonitory change of scaling and other premonitory patterns – *clustering in space-time* and *long-range correlations*. They have been found in spatially extended systems, real and numerically simulated (e.g., Keilis-Borok, 1990; Romanowicz, 1993; Press and Allen, 1995; Sornette and Sammis, 1995; Aki, 1996; Bowman *et al.*, 1998; Pollitz *et al.*, 1998; Keilis-Borok and Shebalin, 1999; Turcotte *et al.*, 2000). The branching diffusion model defines these patterns through a small number of common control parameters, thus liberating predictability studies from a major millstone.

2. Data Analysis

2.1. *Definitions*

– Prediction is targeted at extreme point events with occurrence times T_e, $e = 1, 2, \ldots$

– Premonitory patterns are looked for in an observable time process *S(t)*, hypothetically containing such patterns. In many problems this process is defined as a time series (t_i, m_i, g_i), $i = 1, 2, \ldots$ Here t is the time of the event, $t_i, \leq t_{i+1}$; m is its size (often given in logarithmic scale), g stands for additional parameters that might be indicated (e.g., vector of coordinates of earthquake's hypocenter).

– Scaling of a process *S(t)* is a function $N^S(m)$ – the number of events of the size $\geq m$. We consider its normalized form equivalent to probabilistic distribution function: $P^S(m) = N^S(m)/\check{N}^S$. Here $\check{N}^S$ is the total number of events considered; by definition this is the ordinate of the left end of that curve (at minimal m).

– In many problems the data consist of some average characteristics of system's behavior. For the socio-economic crises considered here these characteristics are monthly indicators. In such a case scaling is determined for the change of the indicator's trend; its definition follows. $f(t)$ – a monthly indicator. $W^f(t/q)=K^f(t/q)(t-q) + B^f(t/q),\ 0<t<q$. This is the local linear least-squares regression of $f(t)$ within the sliding time window $(t\text{-}q,\ t)$. $S^f(t/s,u) = K^f(t/s) - K^f(t-s/u)$ – an 'event': the change of the trend $K^f(t/q)$ between consecutive intervals: current $(t-s,\ t)$ and previous $(t-s-u,\ t-s)$ intervals. Time and, accordingly, parameters s, u are the integers, measured in the number of months. Size m of events, for which the scaling is determined, is the absolute value of S^f.

– Time considered is divided into periods of three kinds, as shown in Fig. 1 (see page 240). To explore premonitory change of scaling we compare functions $P(M)$, and number of events $\check{N}$ in the periods N and D. In the subsequent text lower indexes identify these periods (e.g., $\check{N}_{\mathrm{N}}$, $\check{N}_{\mathrm{D}}$).

2.2. *Point of Departure: Strong Earthquakes*

Three examples of seismicity analysis are shown in Fig. 2 (see page 240). *Critical transitions* ('prediction targets') are the main shocks with magnitude $M \geq 6.4$; here M is the logarithmic measure of energy released by an earthquake. *Size distribution $P(m)$* is probability that the size of an event is $\geq m$; total number of events is $\check{N}_{\mathrm{N}}=277$and $\check{N}_D=255$ for the periods N and D respectively; *a*: events are individual main shocks; measure of size m is their magnitude; *b, c*: events are clusters of aftershocks formed around individual main shocks (Keilis-Borok *et al.*, 1980; Molchan *et al.*, 1990); measure of cluster's size is number of aftershocks not weighted (*b*) or weighted (*c*) by their magnitudes.

2.3. *Socio-Economic Crises*

Three examples of data analysis are shown in Fig. 3 (see page 240). *Critical transitions* ('prediction targets') are the starting points of a respective crisis. *Size distribution $P(m)$* is probability that the size of an event is $\geq m$. Event is the change of a monthly indicator considered: industrial production before recessions (*a*) and unemployment surges (*b*); and monthly rates of lesser crimes – assaults with firearms – before homicide surge (*c*). Total number of events in periods N and D: $\check{N}_{\mathrm{N}}=\check{N}_{\mathrm{D}}=62$ (*a*); $\check{N}_{\mathrm{N}}=24$, $\check{N}_{\mathrm{D}}=44$ (*b*); and $\check{N}_{\mathrm{N}}=21$, $\check{N}_{\mathrm{D}}=28$ (*c*).

2.4. *Modeling*

Similar premonitory change of scaling has been found in a variety of models

– Models of inverse, direct, and colliding cascades (e.g., Allègre *et al.*, 1982, 1995; Narkunskaya and Shnirman, 1994; Gabrielov *et al.*, 2000; Zaliapin *et al.*, 2003).

– Laboratory experiments with fracturing of rocks and metals (Rotwain *et al.*, 1997).

– Models of tectonic blocks and faults (Soloviev and Ismail-Zadeh, 2003).

– Branching diffusion model (Gabrielov *et al.*, 2007).

3. Discussion

– We followed here the 'premonitory patterns' approach in which prediction is targeted at the rare extreme point events. This approach is complementary to classical Kolmogoroff-Wiener prediction, targeted at extrapolation of a whole process, i.e. mainly at the medium or small events.

– That approach is complementary also to cause-and-effect analysis. Extreme events and premonitory patterns often are the parallel manifestations of evolution of the complex system.

– Furthermore premonitory patterns might predict not an extreme event *per se*, but the system's destabilization making it ripe for an extreme event; its triggering then becomes close to inevitable, not requiring a particularly strong impact.

– Transition to predicting individual extreme events comprises at least the following further problems:

Parametrization of premonitory change of scaling, sensitivity analysis, and optimization (Molchan, 2003).

Similar analysis for clustering (e.g., Keilis-Borok *et al.*, 1980) and correlation range (e.g., Shebalin, 2006).

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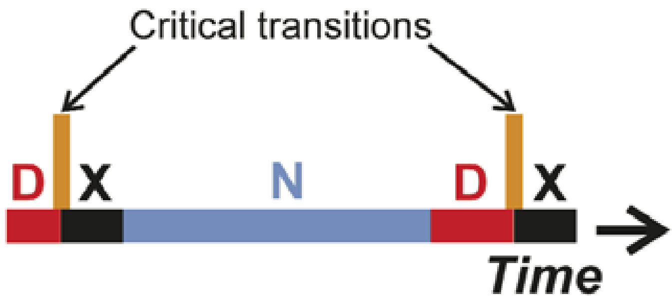

Figure 1. Division of time into periods of three kinds by their relation to critical transitions.

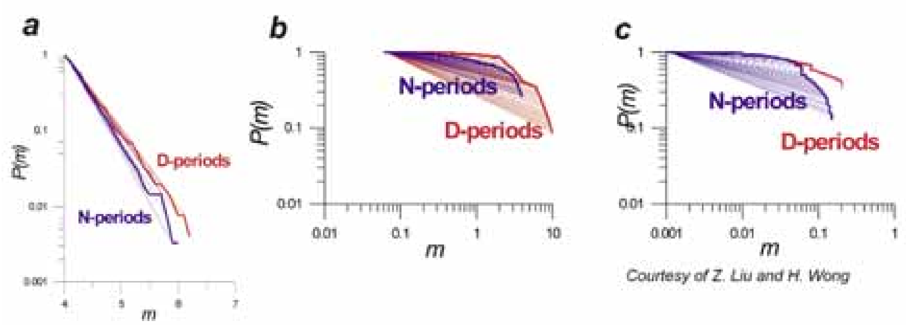

Figure 2. Change of scaling before strong earthquakes. Southern California, 1954-2006 (*a*), California, 1968-2005 (*b, c*).

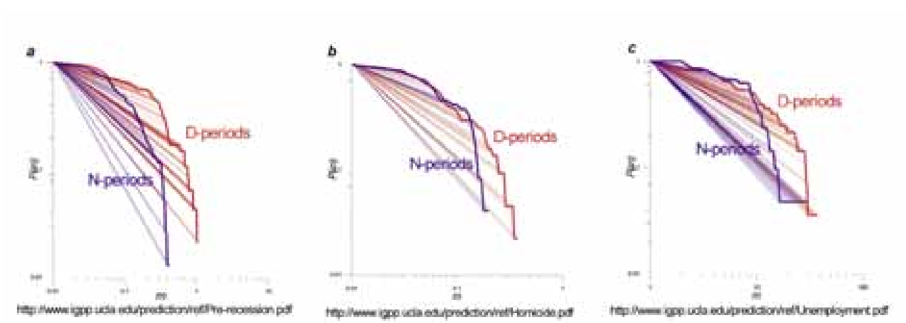

Figure 3. Change of scaling before socio-economic crises. Economic recessions, US, 1961-2002 (*a*); Surges of unemployment, US, 1961-2005 (*b*), Surge of homicides, Los Angeles, 1975-1993 (*c*).